Sullivan
MAC 1105
ch 4 (V2)
practice for the test
(SOLUTIONS)

(1) average rate of change $= \text{slope} = \frac{3}{5}$ (A)

$f(x) = \frac{3}{5}x + 1$

(2) $F(x) = -6 \Rightarrow y = -6$ (horizontal) (A)

so $m = \text{rate of change} = 0$

(3)

x	y
5	10
9	18
13	26
17	34

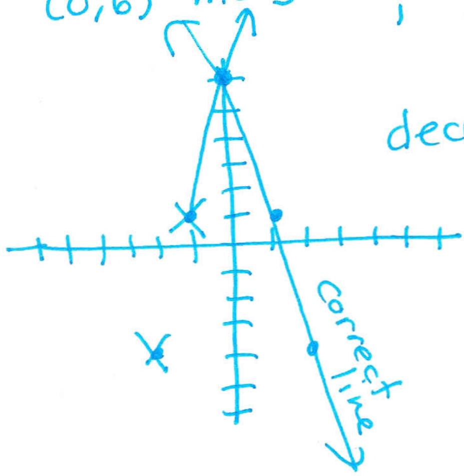
+4, +4, +4 ; +8, +8, +8

$m = \frac{\Delta y}{\Delta x} = \frac{8}{4} = 2$ (A)

so it is linear

(4) $h(x) = -5x + 6$

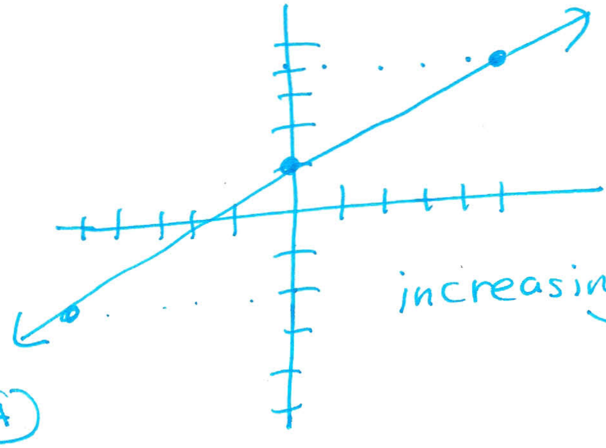
$(0,6)$ $m = -5 = \frac{-5}{1} = \frac{5}{-1}$



decreasing

(5) $f(x) = \frac{3}{5}x + 1$

$(0,1)$ $m = \frac{3}{5} = \frac{-3}{-5}$



increasing

(6) $f(x) = -x - 8$ (B)

$g(x) = x - 15$

a) $f(x) = 0$

$-x - 8 = 0$

$-8 = x$

b) $g(x) = 0$

$x - 15 = 0$

$x = 15$

c) $f(x) = g(x)$

$-x - 8 = x - 15$

$-8 + 15 = x + x$

$7 = 2x$

$\frac{7}{2} = x$

(7) $f(x) = -x - 8$, $g(x) = x - 12$

a) $f(x) > 0$

$-x - 8 > 0$

$-x > 8$

$x < -8$

b) $g(x) > 0$

$x - 12 > 0$

$x > 12$

c) $f(x) \le g(x)$ (C)

$-x - 8 \le x - 12$

$-2x \le -12 + 8$

$-2x \le -4$

$x \ge 2$

8) $\boxed{f(x) = x^2 - 10x + 25}$

shape: ↑↑ $(a=1 \quad b=-10 \quad c=25)$

vertex: (not readable) $= \boxed{(5,0)} = \boxed{(h,k)}$

$h = \frac{-b}{2a} = \frac{-(-10)}{2(1)} = 5$

$k = (5)^2 - 10(5) + 25$
$= 0$

$a = 1 \rightarrow$ + up
1 neutral

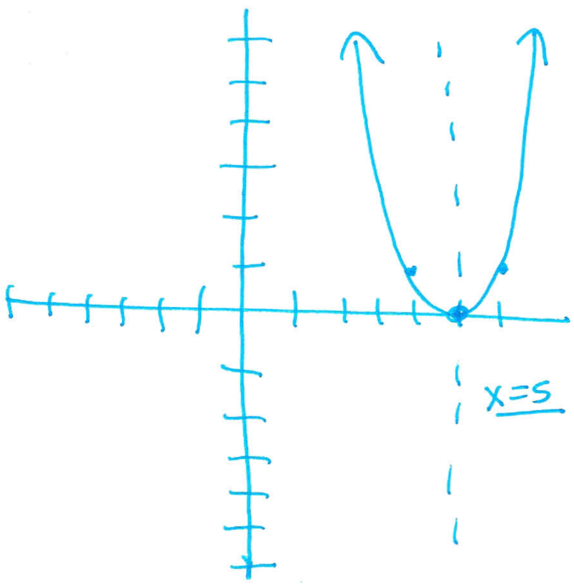
x-intercepts (5,0)

$x = h \pm \sqrt{\frac{-k}{a}} = 5 \pm \sqrt{\frac{0}{1}} = 5$

y-intercept

$x = 0 \quad f(0) = 25$

axis of symmetry
$(x = h)$ $\quad x = 5$



x	y
4	1
5	0
6	1

D: $\mathbb{R}$

R: $y \geq 0$
or $[0, \infty)$

9) $\boxed{f(x) = -x^2 + 4x + 5}$

shape: ↑↑ $(a=-1 \quad b=4 \quad c=5)$

vertex: (not readable)

$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$

$k = -(2)^2 + 4(2) + 5 \quad (2,9)$
$= -4 + 8 + 5 = 9$

$a = -1 \rightarrow$ − down
1 neutral

x-intercepts use $x = h \pm \sqrt{\frac{-k}{a}}$

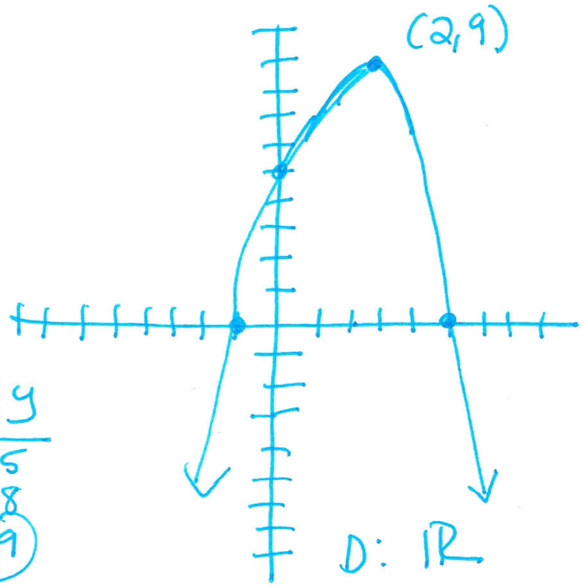
$x = 2 \pm \sqrt{\frac{-9}{-1}} = 2 \pm \sqrt{9}$
$= 2 \pm 3$

$2+3 = 5$
$2-3 = -1$
$\boxed{(5,0) \\ (-1,0)}$

y-intercept $x = 0$

$f(0) = -(0)^2 + 4(0) + 5$
$= 5$ so $\boxed{(0,5)}$

axis of symmetry $x = h$
so $x = 2$



x	y
0	5
1	8
2	9
3	8
4	5

D: $\mathbb{R}$

R: $y \leq 9$
$[-\infty, 9]$

10 $\boxed{f(x) = x^2 + 8x + 7}$

$\boxed{a = 1 \quad b = 8 \quad c = 7}$

shape: ↻

vertex: (not readable) $= \boxed{(-4, -9)}$
(h, k)

$$h = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$$

$$k = (-4)^2 + 8(-4) + 7 = -9$$

$a = 1$ + up
           1 neutral

x-intercepts $x = h \pm \sqrt{\frac{-k}{a}}$ $(y=0)$

$$x = -4 \pm \sqrt{\frac{-(-9)}{1}} = -4 \pm 3$$

$\Rightarrow -4+3 = -1$
$\quad\;\; -4-3 = -7$

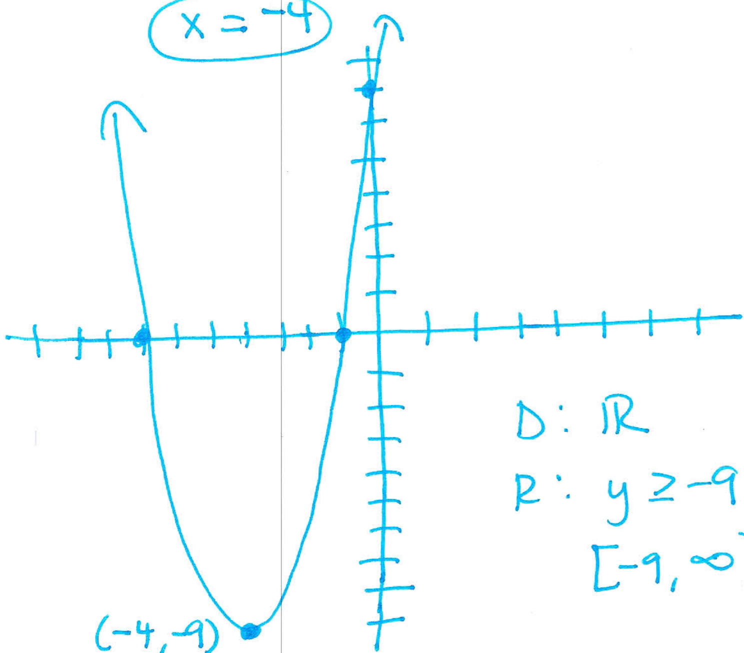
$\boxed{(-1, 0) \quad (-7, 0)}$

y-intercept $(x=0)$

$f(0) = 7$

axis of symmetry $x = h$

$x = -4$



D: $\mathbb{R}$

R: $y \geq -9$

$[-9, \infty)$

11 $\boxed{f(x) = -5x^2 - 3}$

$a = -5 \quad b = 0$
$c = -3$

shape: ↻

vertex: readable

$y = -5(x - 0)^2 - 3$ $\boxed{(0, -3)}$ $\boxed{(h, k)}$

$a = -5$ − down
           5 stretched (narrow)

x-intercepts

$$x = h \pm \sqrt{\frac{-k}{a}}$$

$$x = 0 \pm \sqrt{\frac{-(-3)}{-5}}$$

$$= 0 \pm \sqrt{\frac{-3}{5}}$$ ← not real

$\Rightarrow$ no x-intercepts

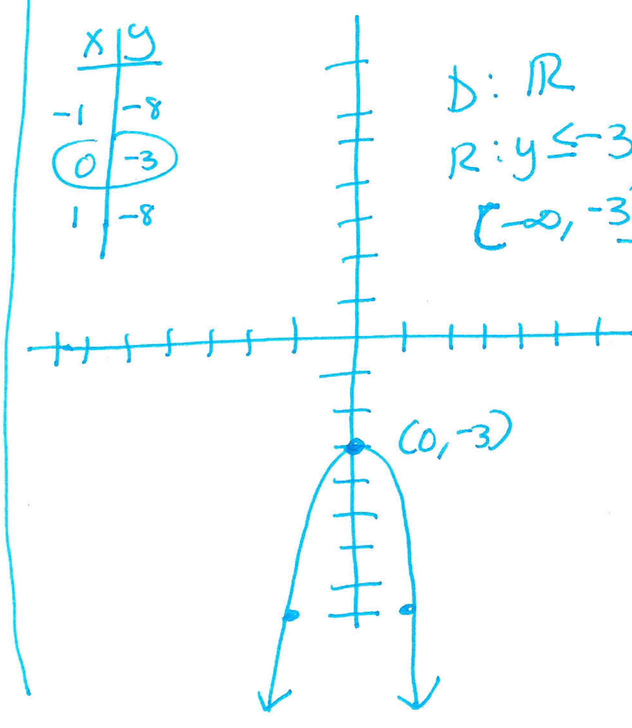
y-intercepts $(x=0)$

$f(0) = -3$ $\boxed{(0, -3)}$

axis of symmetry

$x = 0$ (the y axis!)

x	y
-1	-8
0	-3
1	-8



D: $\mathbb{R}$

R: $y \leq -3$

$[-\infty, -3]$

(12) $\boxed{f(x) = \frac{1}{4}x^2 + 3}$

$a = \frac{1}{4} \quad c = 3$

$f(x) = \frac{1}{4}(x-0)^2 + 3$

shape: ↑∪↑

vertex: readable $(0, 3)$

$a = \frac{1}{4}$ $\quad \frac{1}{4}$ compressed (wide)

$+$ up

x-intercept $\quad x = h \pm \sqrt{\frac{-k}{a}}$

$x = 0 \pm \sqrt{\frac{-3}{\frac{1}{4}}}$

$x = \pm\sqrt{\frac{-3}{1} \cdot \frac{4}{1}} = \pm\sqrt{-12}$

not real

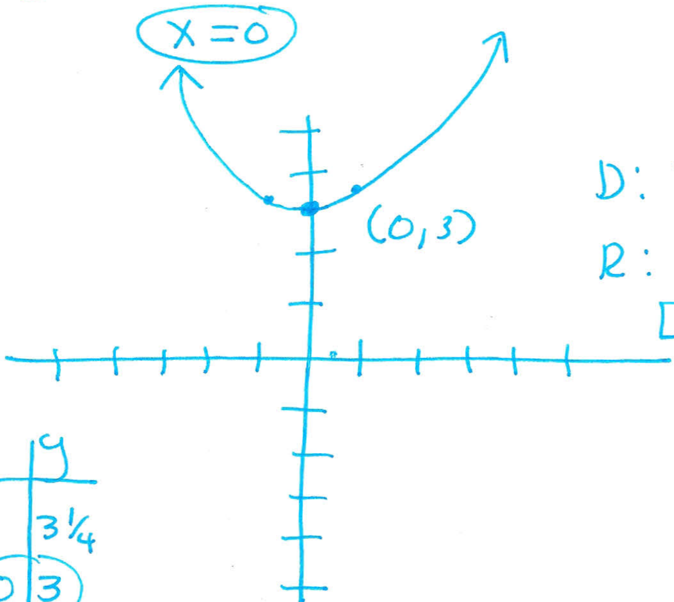
$\boxed{\text{no x-intercepts}}$

$\boxed{\text{y-intercepts}}$ $\quad x = 0 \quad \boxed{(0,3)}$

$f(0) = \frac{1}{4}(0)^2 + 3 = 3$

axis of symmetry $\quad x = h$

$x = 0$



D: $\mathbb{R}$

R: $x \geq 3$

$[3, \infty)$

x	y
-1	$3\frac{1}{4}$
0	3
1	$3\frac{1}{4}$

(13) $\boxed{f(x) = -3x^2 + 12x}$

$a = -3 \quad b = 12 \quad c = 0$

$h = \frac{-b}{2a} = \frac{-12}{2(-3)} = 2$

$k = -3(2)^2 + 12(2)$

$= -3(4) + 24 = 12$

$\boxed{(2, 12)}$

axis of symmetry $\quad x = h$

$x = 2$

(14) $\boxed{f(x) = x^2 + 8x}$

$a = 1 \quad b = 8 \quad c = 0$

$h = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$

$k = (-4)^2 + 8(-4) = -16$

$16 - 32$

$\boxed{(-4, -16)}$

$\boxed{x = -4}$

(15) $\boxed{f(x) = -10x^2 - 2x - 3}$

$a = -10 \quad b = -2 \quad c = -3$

$h = \frac{-b}{2a} = \frac{-(-2)}{2(-10)} = \frac{-1}{10}$

$k = -10\left(\frac{-1}{10}\right)^2 - 2\left(\frac{-1}{10}\right) - 3$

$= -10\left(\frac{1}{100}\right) + \frac{2}{10} - 3$

$= -\frac{1}{10} + \frac{2}{10} - \frac{30}{10}$

$= \frac{-29}{10}$

$\boxed{\left(\frac{-1}{10}, \frac{-29}{10}\right) \quad x = \frac{-1}{10} \text{ axis of symmetry}}$

(16) $\boxed{f(x) = 3x^2 + 3x - 9}$

min or max = vertex

$h = \frac{-b}{2a} = \frac{-3}{2(3)} = \frac{-1}{2}$

$k = 3\left(\frac{-1}{2}\right)^2 + 3\left(\frac{-1}{2}\right) - 9 = \frac{-39}{4}$

$(h, k) = \left(\frac{-1}{2}, \frac{-39}{4}\right)$

up ward $\rightarrow$ min at $-39/4$ (the k value)

(17) $\boxed{f(x) = -2x^2 + 2x}$

$h = \frac{-b}{2a} = \frac{-2}{2(-2)} = \frac{1}{2}$

$k = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) = \frac{1}{2}$

$= -2\left(\frac{1}{4}\right) + 1$

$(h, k) = \left(\frac{1}{2}, \frac{1}{2}\right)$

downward $\Rightarrow$ max at $\frac{1}{2}$ (the k value)

(18) $\boxed{f(x) = -x^2 - 3x - 9}$

$h = \frac{-b}{2a} = \frac{-(-3)}{2(-1)} = \frac{-3}{2}$

$k = -\left(\frac{-3}{2}\right)^2 - 3\left(\frac{-3}{2}\right) - 9 =$ min at $\frac{-27}{4}$

$= -\left(\frac{9}{4}\right) + \frac{9}{2} - 9$

$= \frac{-9}{4} + \frac{18}{4} - \frac{36}{4}$

(19) $(h, k) = (-2, 1)$

$(x, y) = (0, 5)$

$y = a(x-h)^2 + k$

$y = a(x+2)^2 + 1$

$(x, y) \rightarrow (0, 5)$ to find a

$5 = a(0+2)^2 + 1$

$4 = a(4)$

$1 = a$

so $\boxed{y = 1(x+2)^2 + 1}$

(20) lowest cost $\Rightarrow$ vertex

$C(x) = 2x^2 - 32x + 600$

$h = \frac{-(-32)}{2(2)} = 8$

$k = 2(8)^2 - 32(8) + 600$

$= \$472$

lowest cost occurs when 8 videos are rented. The cost is \$472

C

21. DONE ON LAST PAGE
Challenge Problem!

22. $R(p) = -5p^2 + 1120p$

maximize Revenue → find the vertex!

(p) $h = \frac{-b}{2a} = \frac{-1120}{2(-5)} = \$112$

(R) $k = -5(112)^2 + 1120(112)$
$= \$62{,}720$ total Revenue

C

25. $P = c - \left(\frac{c-s}{L}\right)t$

$C = 20{,}000$
$S = 5000$
$L = 6$ (6 useful years)
$t = 7$

$p = 20000 - \left(\frac{20000 - 5000}{6}\right)7$

$p = \$2500$

D

23. $P(x) = -0.004x^2 + 2.8x - 250$

maximize profit → find the vertex!

(x) $h = \frac{-2.8}{2(-0.004)} = 350$ pretzels

(P) $k = -0.004(350)^2 + 2.8(350) - 250$
$= \$240$ profit

A

24. $h(t) = -16t^2 + 40t + 50$

max height ⇒ vertex

(t) $h = \frac{-b}{2a} = \frac{-40}{2(-16)} = 1.25$ s.

A

(h) height $k = -16(1.25)^2 + 40(1.25) + 50$
$= 75$ ft max height

total time to hit the ground
$2 \times 1.25s = 2.5s$

26.

Revenue $= R(x)$
Profit $= P(x)$
Cost $= C(x)$

$P(x) = R(x) - C(x)$

still 26 x = # of manicures

$R(x) = 12x$

$C(x) = 7.35x + 120$

variable cost, fixed cost

$P(x) = 12x - (7.35x + 120)$
$= 12x - 7.35x - 120$
$P(x) = 4.65x - 120$

$P(200) =$
$= 4.65(200) - 120$
$= \$810$

31. Equilibrium price
supply = demand

$S(p) = 4830 - 80p$
$D(p) = 130p$

$4830 - 80p = 130p$
$4830 = 210p$
$23 = p$

C

$S(23) = 4830 - 80(23) = \$2990$
also $D(23) = 130(23) = \$2990$

so you can use either $D(p)$ or $S(p)$